# FLEXURAL CRACK ANALYSIS IN REINFORCED CONCRETE BEAMS WITH SHORT SHEAR SPAN LENGTH

**Rendy Thamrin (rendy@ft.unand.ac.id)**
Noor Azlina Abdul Hamid (azlinah@uthm.edu.my), Zalipah Jamellodin (zalipah@uthm.edu.my)
Muhammad Aminsyah (aminsyah@ft.unand.ac.id), Riza Aryanti (riza@ft.unand.ac.id)

## ABSTRACT

This paper presents experimental study focused on flexural crack in reinforced concrete beams with short span length. Four simple supported beams loaded monotonically with four-point bending were tested until failure. The test variables in this study were: 1) ratio of longitudinal reinforcement, and 2) ratio of transversal reinforcement. Analytical study based on layer element method was also carried out to obtain the progress of flexural crack and to obtain complete response of the beams analytically. Four main region can be recognized from the crack depth path during the loading history obtained from the test. Moreover, test results show that ratio of longitudinal reinforcement influences significantly the flexural capacity of the beam as well as the progress of flexural crack. In addition, the ratio of transversal reinforcement also influences the beam capacity and slightly affects the progress of flexural crack. Furthermore, the analytical prediction of flexural crack were also compared to the test results and the comparison shows a good agreement each other.

**KEY WORDS:** reinforced concrete beams, flexural crack, layer element method, crack depth

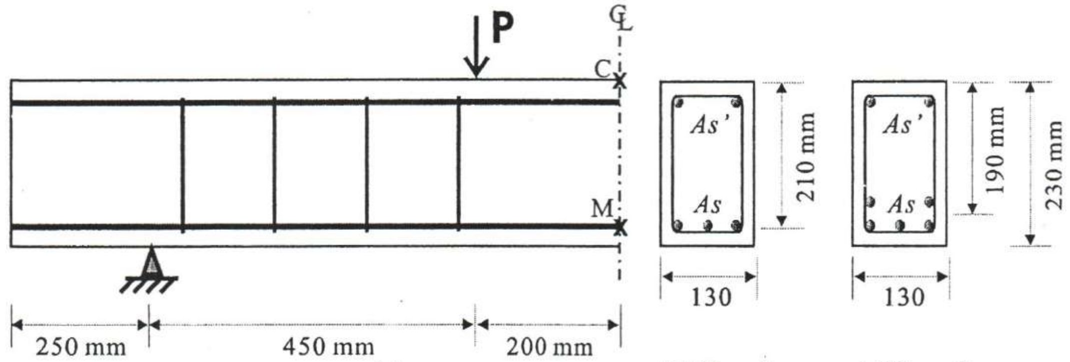
## 1. INTRODUCTION

Crack in reinforced concrete structure is one of important parameter that should be checked during the design process. The codes also provide crack limitation due to aesthetical and structural protection reason (ACI 318M-08, SNI-03-2847-2002, BS-8110, BS 8110-1-1997). The behavior of crack growth in reinforced concrete strongly influences by the level of stress in longitudinal reinforcement (Nilson, et.al.2004). The crack depth also can be used to characterize the level of deterioration of reinforced concrete structures. For this explanation, the investigation of crack growth in reinforced concrete structures is very important to carry out.

In this experimental study the flexural capacity of reinforced concrete beams was observed and two test variables were used in this study i.e.: 1) ratio of longitudinal reinforcement, and 2) ratio of transversal reinforcement. The growth of neutral axes depth and crack depth was also investigated experimentally. This study not only presents the experimental results but also analytical study to obtain a complete response of loading history of reinforced concrete beam. The layer element method was used in analytical study and a computer program was developed to facilitate the calculation process.

## 2. EXPERIMENTAL STUDY

In this experimental study, totally four reinforced concrete beams were designed to study the crack behavior under monotonic load. The beam specimens were consisted of two type of cross section as shown in Figure 1. All of beam specimens were subjected to four point bending and loaded monotonically until failure using a hydraulic actuator with 1000 kN capacity. The dimension of beam cross section was 130 mm wide and 230 mm deep (see Figure 1). The short shear span length of 450 mm (a/d<2.5) was used and two type of spacing (50 mm and 120 mm) of transversal reinforcement were used. The beams were designed with sufficient anchorage length (250 mm) to avoid bond failure. The detail of beam reinforcement for all specimens are listed in Table 1.



(a) (b) Type I (c) Type II

**Figure 1.** Beam dimension, position of loads, and beam cross section.

**Table 1.** Detail of beam reinforcements

| Specimens | Shear Reinforcement | | | Longitudinal Reinforcement (Tension) | | | Longitudinal Reinforcement (Compression) | | | Flexural Strength Analytic | Flexural Strength Exp. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | s (mm) | $d_s$ (mm) | $fy$ MPa | N | $d_b$ (mm) | $As$ ($mm^2$) | N | $d_b$ (mm) | $As'$ ($mm^2$) | (kN) | (kN) |
| BS-01 | 120 | 8.0 | 700.0 | 3 | 10.0 | 235.6 | 2 | 10.0 | 157.1 | 56.0 | 55.0 |
| BS-02 | 50 | | | | | | | | | 56.0 | 51.0 |
| BS-03 | 120 | | | 5 | | 392.7 | | | | 80.0 | 80.0 |
| BS-04 | 50 | | | | | | | | | 80.0 | 79.0 |

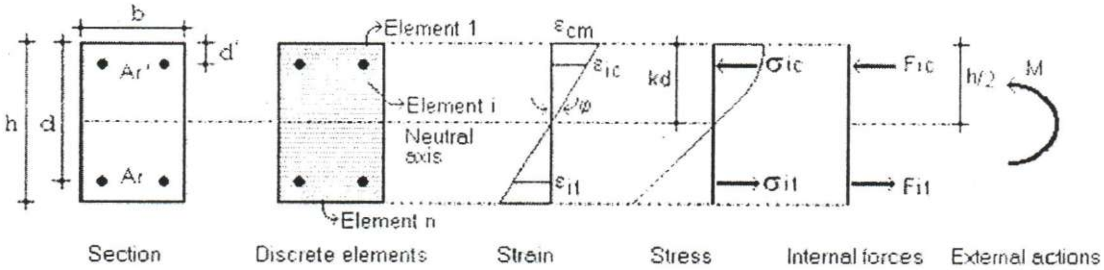


**Figure 2.** Layer element model used in analytical study.

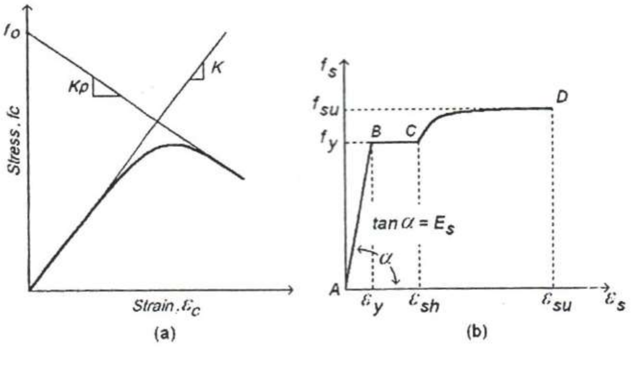


**Figure 3.** Stress-strain model for concrete and steel used in analytical study.

In addition, all of the beams were equipped with strain gauges to measure strain at selected positions showed by cross marks in Fig. 1. Positions of strain gauges attached on longitudinal reinforcement are shown in Fig. 1 and denoted as M. Concrete strain gauges were also attached on the top of compression zone indicated as C at middle point of the beam. Also, three displacement transducers were used to measure deflections at midspan and loading points.

The concrete compressive strength of the beams were 35 MPa and ready-mixed concrete was used. The main longitudinal reinforcement was conventional steel bars with nominal diameter of 10 mm. From the tensile test, elastic modulus of 200 GPa, yield stress of 540 MPa were obtained. In addition, for both type of cross section, steel bars were used as transverse reinforcement and the elastic modulus of 265 GPa and yield stress of 700 MPa were determined from the test.

## 3. ANALYTICAL STUDY

The layer element method (see Figure 2) was applied in this study to obtain a complete response of loading history. The beam cross-section is divided into finite layer element. Strain and material properties of each elements during the loading history were determined throughout incremental strain procedure. The strain compatibility, equilibrium conditions, and constitutive law should be considered in the calculation process. Incremental curvature was applied in order to find the complete response of loading history until the defined maximum value of concrete compressive strain.

Figure 3 illustrated stress-strain relationship of concrete in compression adopted from Almusallam, et.al, 1995. While, stress-strain relationship with strain hardening was applied for steel reinforcement (Esmaeily, A., and Xiao, Y., 2005).

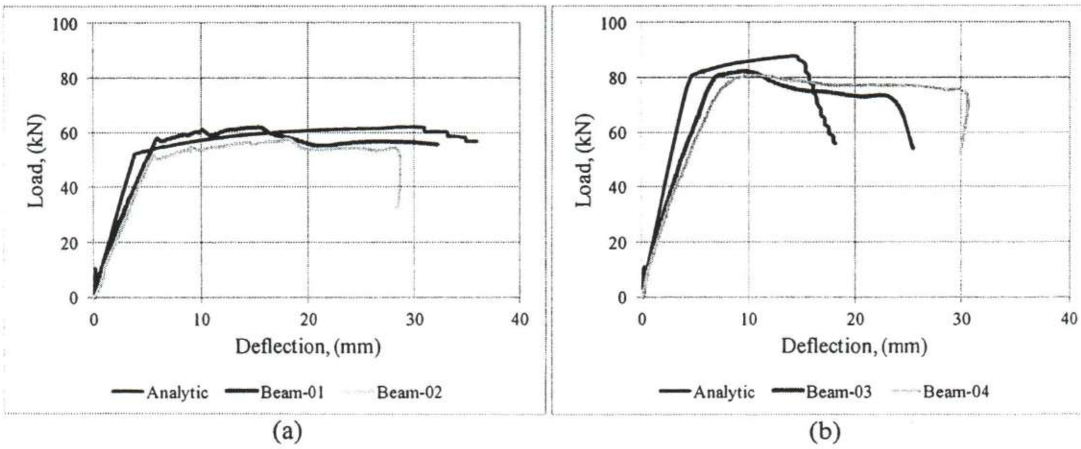
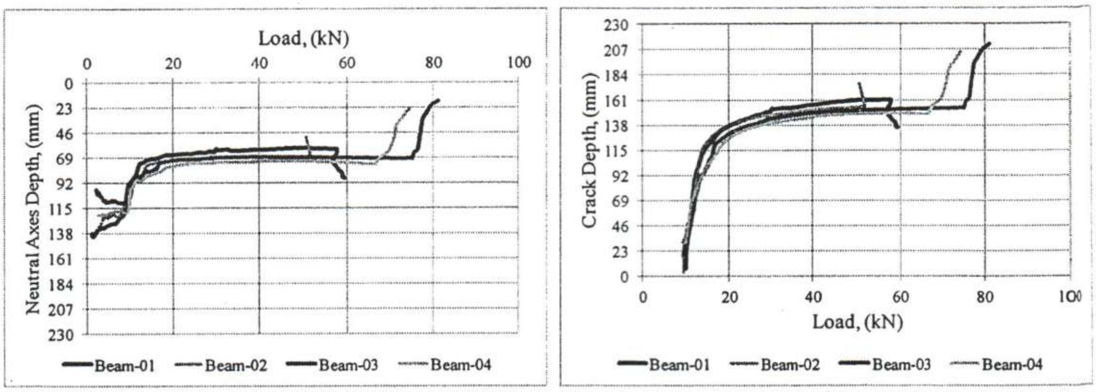


**Figure 4.** Analytical and experimental load vs deflection curves.



(a) (b)

**Figure 5.** Experimental neutral axes depth vs load and crack depth vs load curves.

Estimated deflections at midspan of the beam for each incremental load can be calculated by integrating the curvatures along the member and given by:

$$\delta = \int_A^B x\varphi dx \tag{1}$$

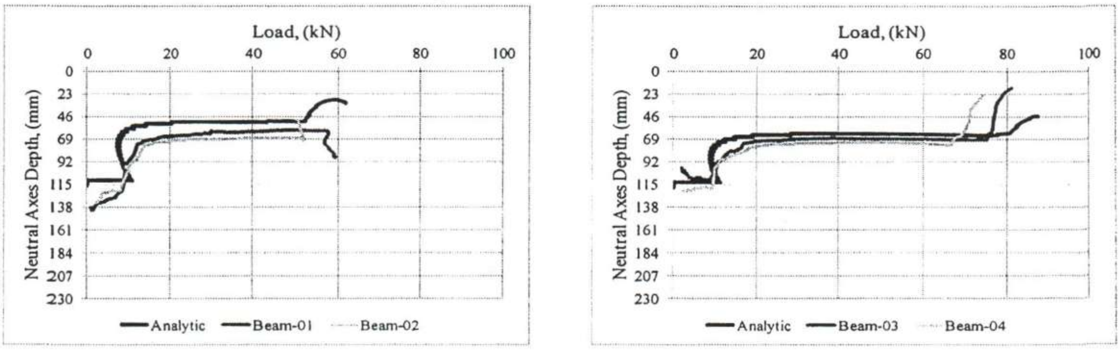
where: A and B = the distance from deflected point to beam support and $\varphi$ = curvature of section corresponding to each load level.

## 4. RESULTS AND DISCUSSIONS

The beam capacities obtained from the test and analytical procedure are listed in Table1. All of the beams were failed in flexural mode indicated by concrete crushing at the top of compression fiber in the zone between two point load. Also, the tensile reinforcements were yielded with large amount of tensile strain. The test results in terms of load-deflection curves are shown in Figure 3. It is illustrated from this figure that beam capacities with higher ratio of transversal reinforcement show higher flexural capacity. Meanwhile beam with higher longitudinal reinforcement ratio exhibit higher flexural capacity.

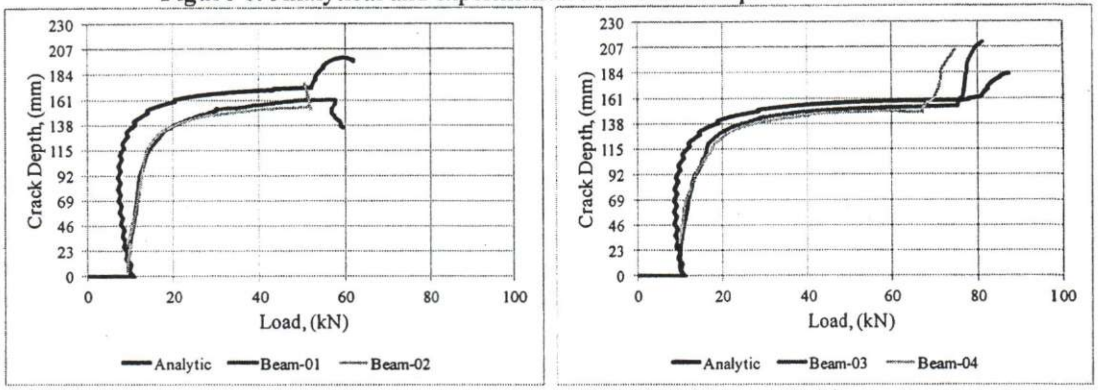
However, Figure 3 also demonstrate that deformation ductility of beams with higher longitudinal reinforcement ratio is lower than beams with lower ratio.

Figure 4 shows the progress of neutral axes depth (see Figure 4(a)) and crack depth during loading history(see Figure 4(b)). It can be seen from Figure 4(a) that the progress of neutral axes depth starts growing up from the middle zone of beam height. Then, after flexural crack occur the slope of the curve become flat until the tension reinforcement reach the yielding point. Furthermore, in case of beam with higher longitudinal reinforcement ratio the curve again raise up until the load stopped.



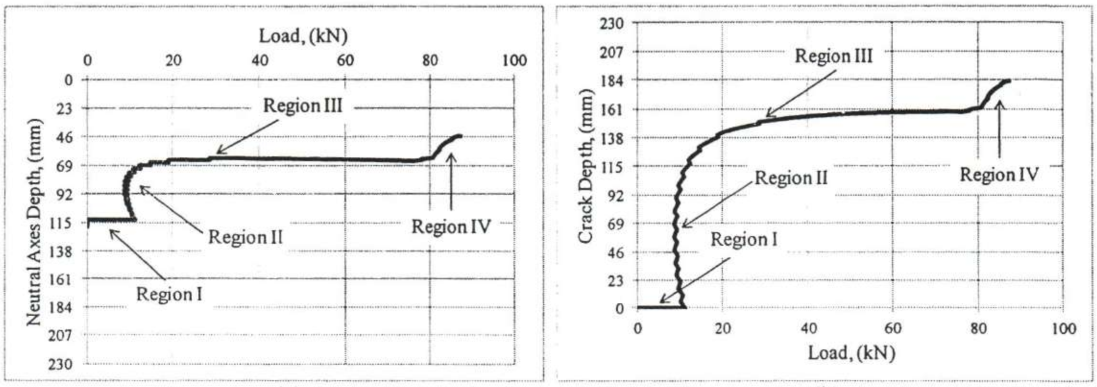
(a) (b)

**Figure 6.** Analytical and experimental neutral axes depth vs load curves.



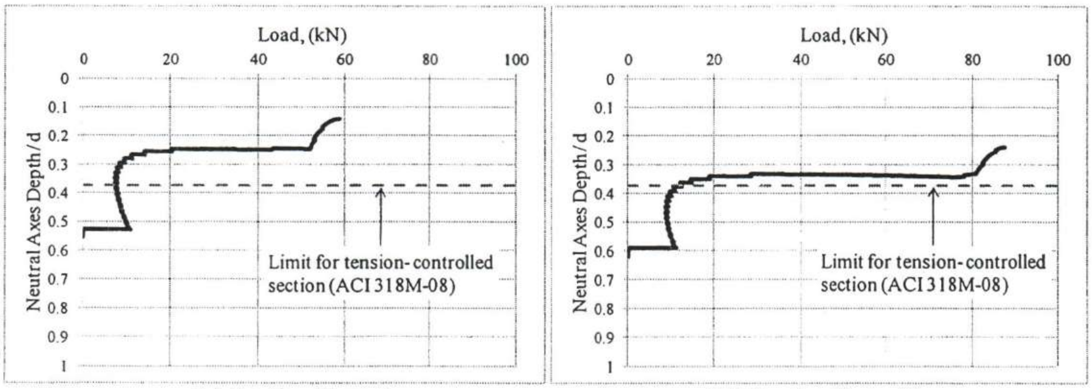
(a) (b)

**Figure 7.** Analytical and experimental crack depth vs load curves.



(a) (b)

**Figure 8.** Analytical neutral axes depth vs load and crack depth vs load curves (section type II).



(a) Section Type I (b) Section Type II

**Figure 9.** Analytical neutral axes depth/d vs load.

Figure 4(b) shows the growth of crack depth during loading history. It is shown that the crack depth of beam with higher longitudinal reinforcement ratio is higher than that beam with lower longitudinal reinforcement ratio.

The comparison between analytical load-deflection curve with experimental is shown in Figure 3. It can be seen that analytical prediction can predict the test result in a good accuracy. Moreover, analytical neutral axes and crack depth are plotted in Figure 5 and Figure 6 together with experimental results. The crack depth in analytical study is determined using Equation (2), (Almusallam, T. H., 1997). It is shown that analytical prediction can predict well the experimental results.

$$yc = (h - c)*(\varepsilon s - (fcr/Ec))/ \varepsilon s) \quad (2)$$

where: $yc$ is the crack depth, $h$ is the beam height, $c$ is the neutral axes depth, $\varepsilon s$ is the tensile strain in longitudinal reinforcement, $fcr$ is the tensile crack stress, and $Ec$ is the modulus elasticity of concrete.

Finally, from the results of experimental and analytical study, it is obtain that four main region can be identified from the neutral axes and crack depth path during the loading history as shown in Figure 7. The first region is zone before the first flexural crack, the second is zone after flexural crack in which the beam in condition to find its equilibrium, the third is zone flat zone before yielding of tension reinforcement and, the fourth is zone after yielding of tension reinforcement.

The limit for neutral axes per effective depth is plotted in Figure 8 together with neutral axes depth vs load curves. It is shown that as the reinforcement ratio increases, the neutral axes depth become closer to the limit line. Mean that the failure mode move down to the transition and compression zone due to higher tensile reinforcement ratio (ACI 318M-05).

## 5. CONCLUSIONS

Four reinforced concrete beams were tested to study the progress of flexural crack. Analytical study was also carried out to obtain the flexural capacity and the progress of flexural crack depth. From the results, the following conclusions are noted:

1. The flexural capacity of beams with higher longitudinal reinforcement ratio is higher than beams with lower longitudinal reinforcement ratio.
2. Beams with higher longitudinal reinforcement ratio show lower displacement ductility.
3. The crack depth of beam with higher longitudinal reinforcement ratio is higher than that beam with lower longitudinal reinforcement ratio.
4. Four main region can be identified from the neutral axes and crack depth path during the loading history.
5. Analytical prediction can predict well the experimental results in a good accuracy.

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